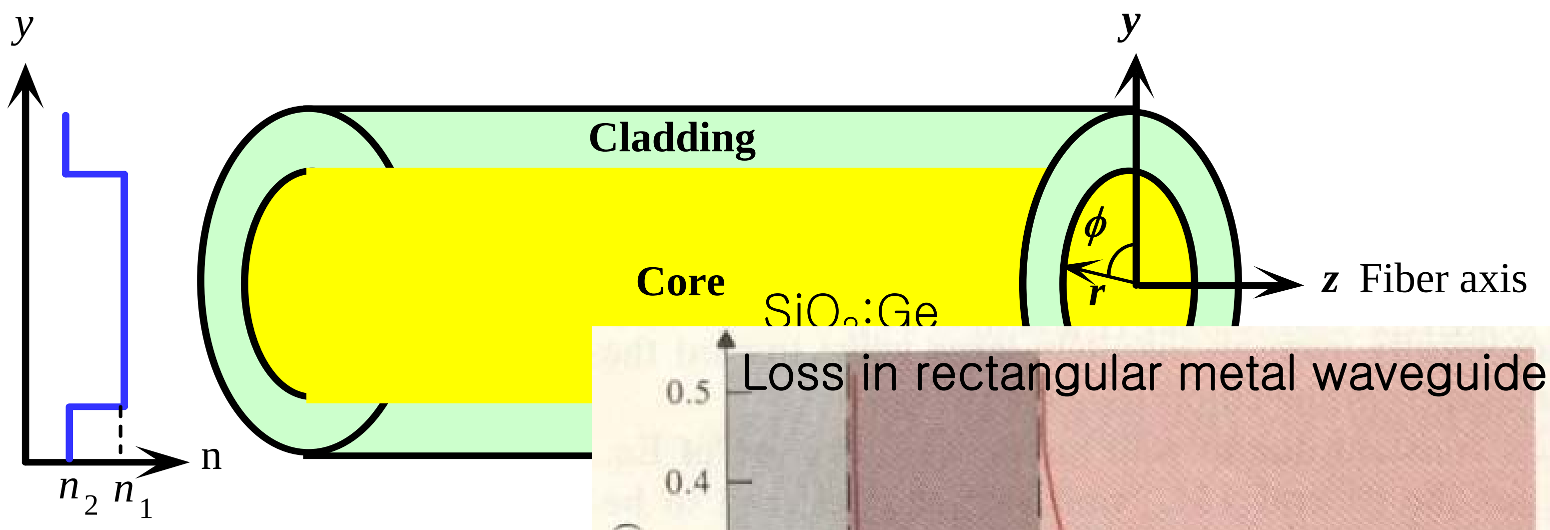
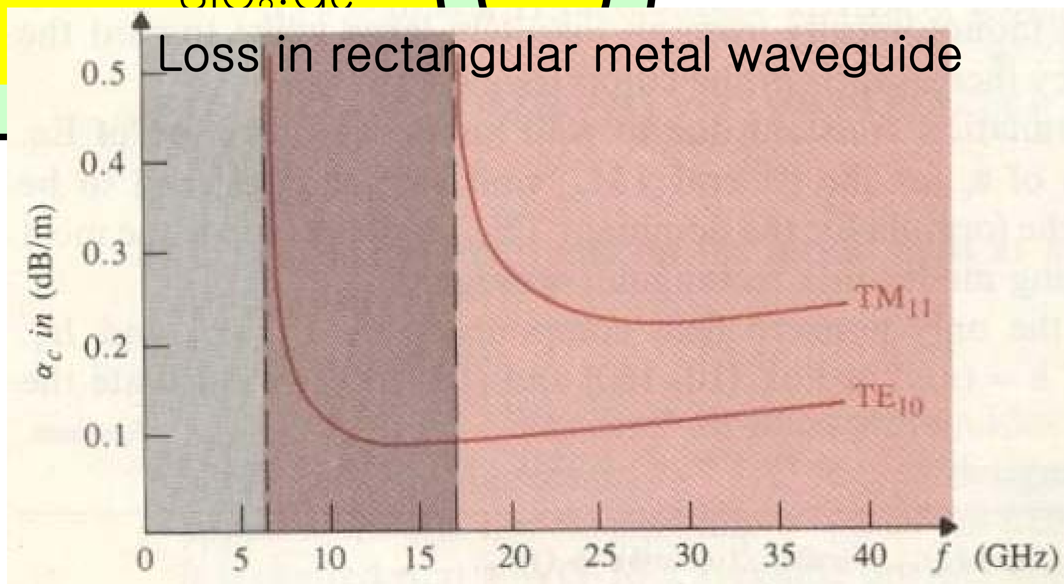
Waveguide Basics

Optical Fiber: Circular dielectric waveguide made of silica (SiO_2)

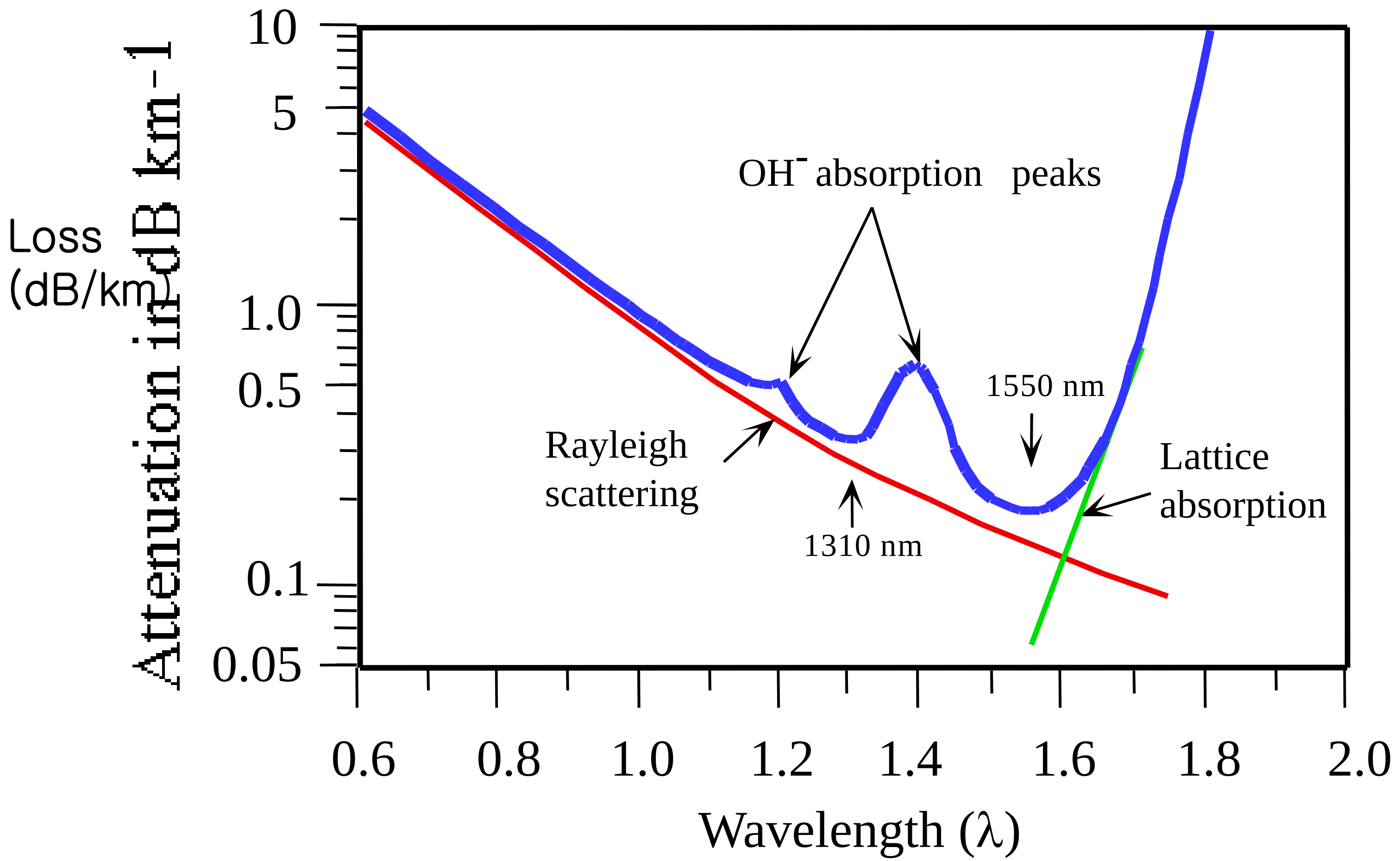




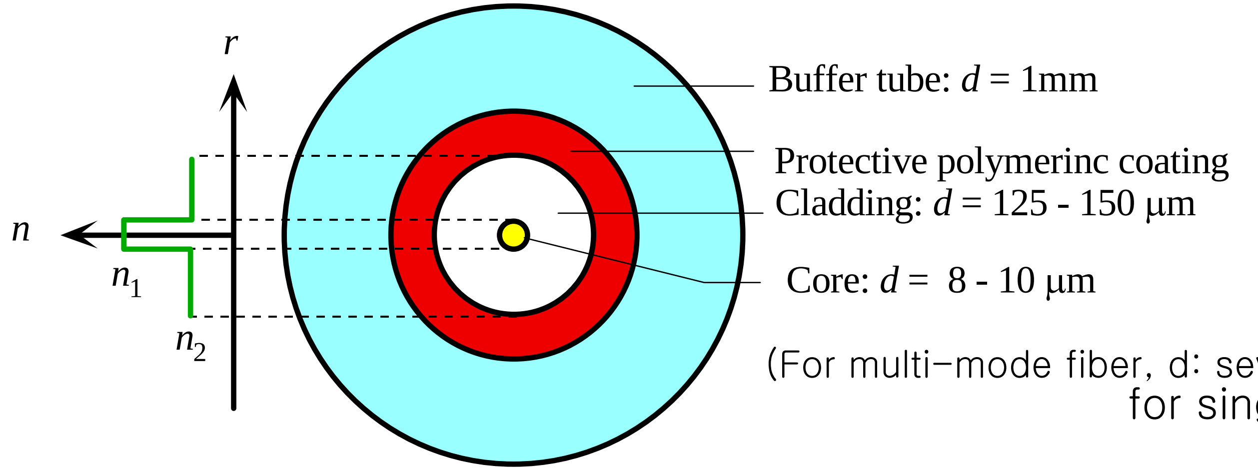
What is special about fiber?
- Extremely low loss: 0.2dB/km
- Can be very long: 100's of km

Waveguide Basics

Loss in fiber

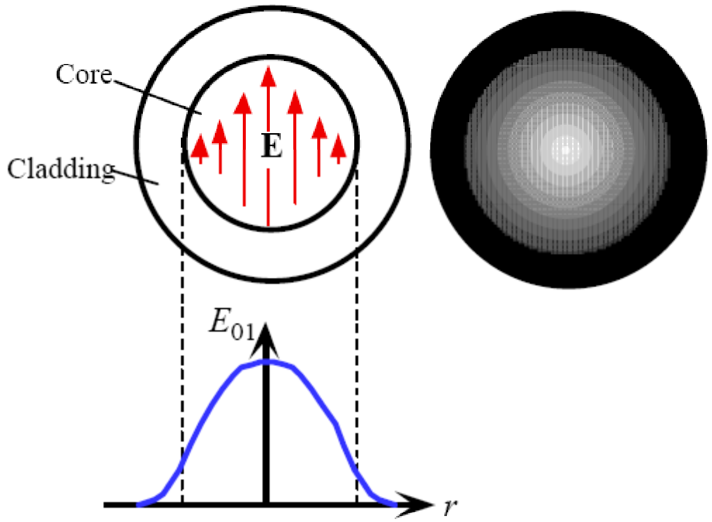


Waveguide Basics



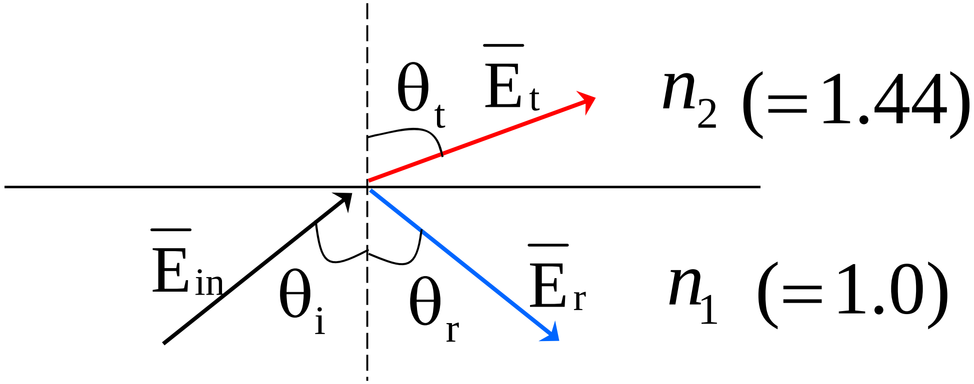
(For multi-mode fiber, d: several tens of μm)
for single-mode fiber

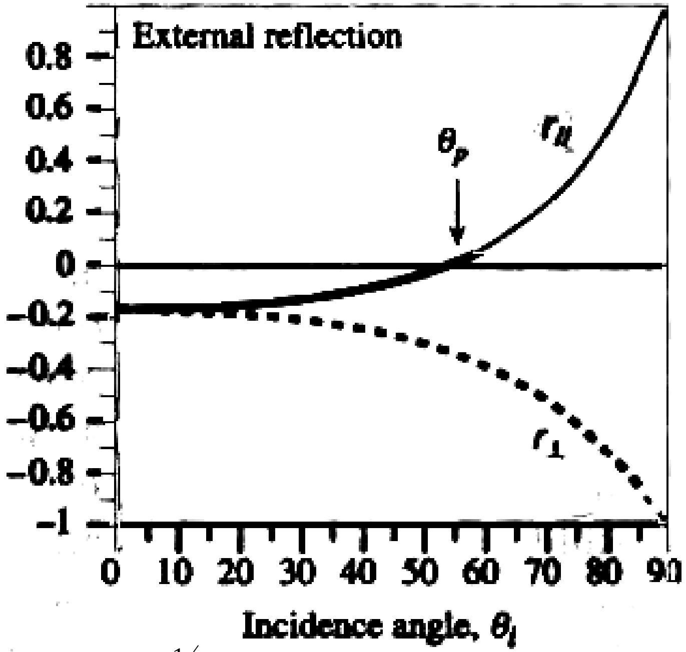
Optical Power Distribution
In Single-**Mode** Fiber



What confines light inside core?

Waveguide Basics





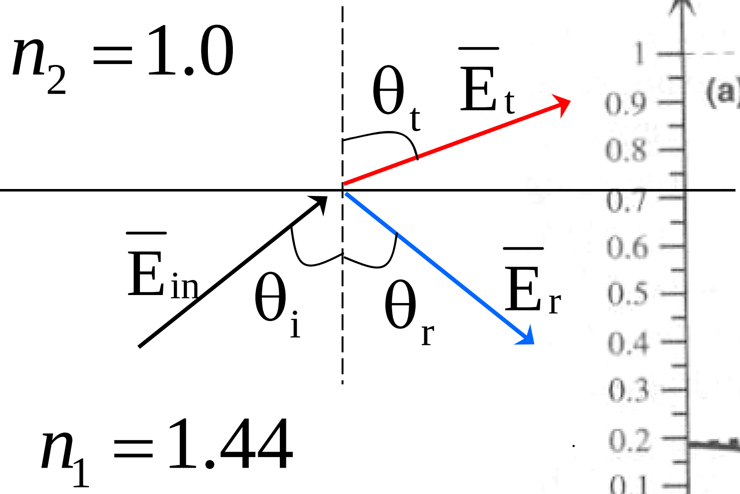
$$r_\perp = \frac{E_r}{E_i} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

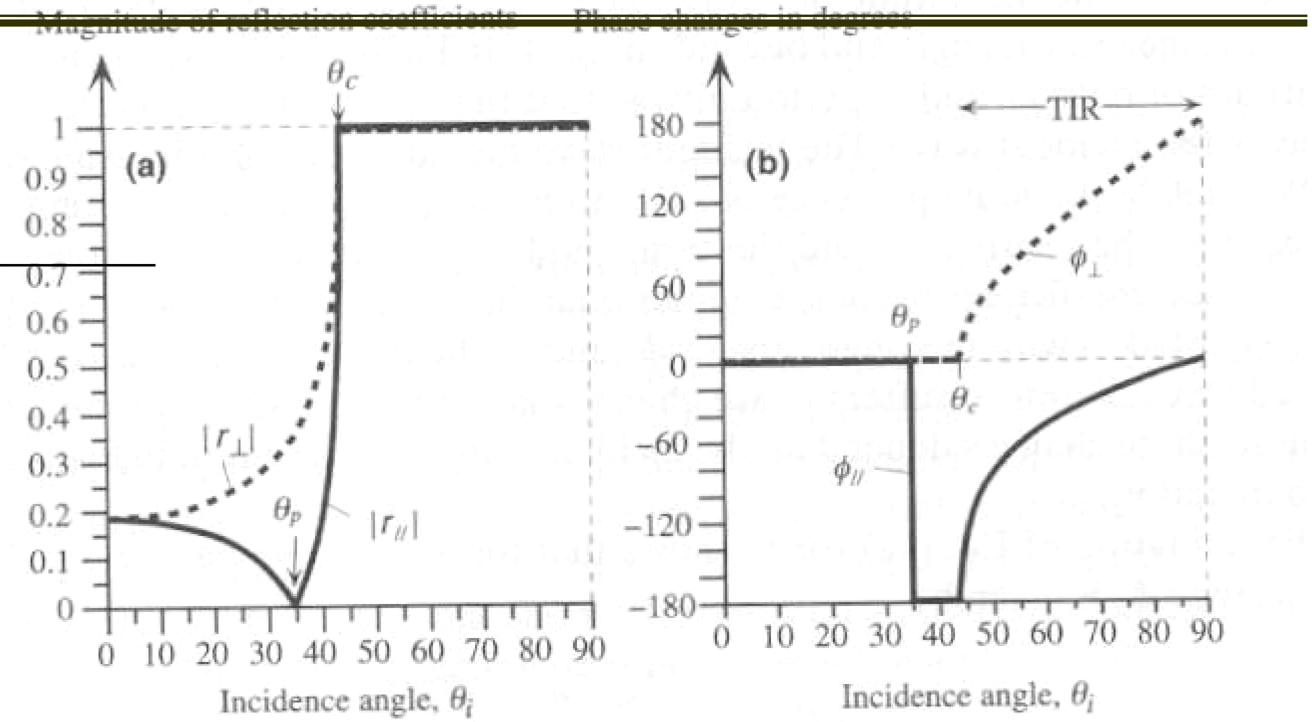
$$r_\parallel = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_\perp = \frac{E_t}{E_i} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad (n = \frac{n_2}{n_1})$$

$$t_\parallel = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

Waveguide Basics



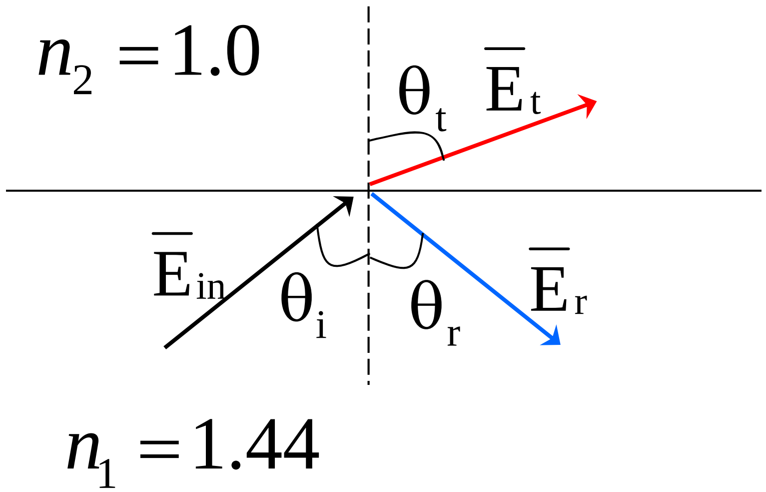


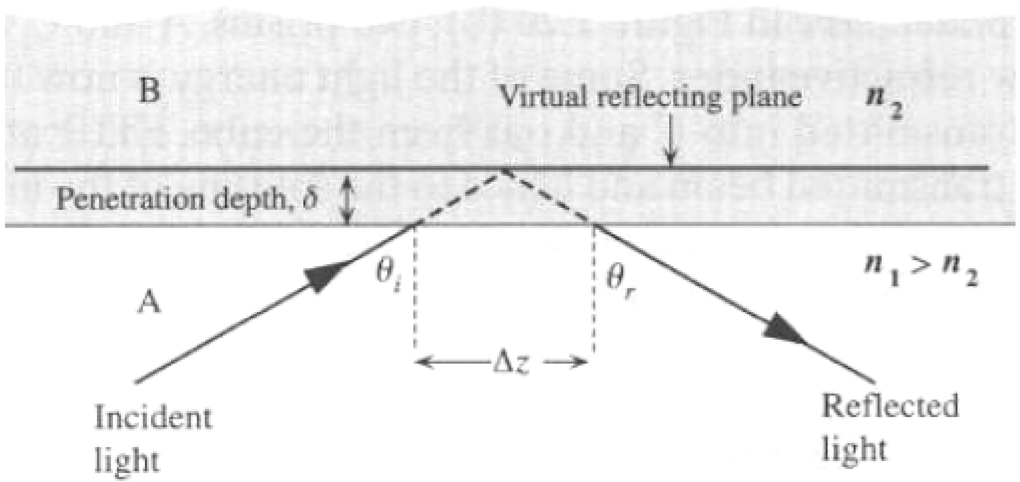
What happens at $\theta_c = \sin^{-1} n$?

$$r_\perp = \frac{E_r}{E_i} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

r becomes complex !
(magnitude 1, phase shift)

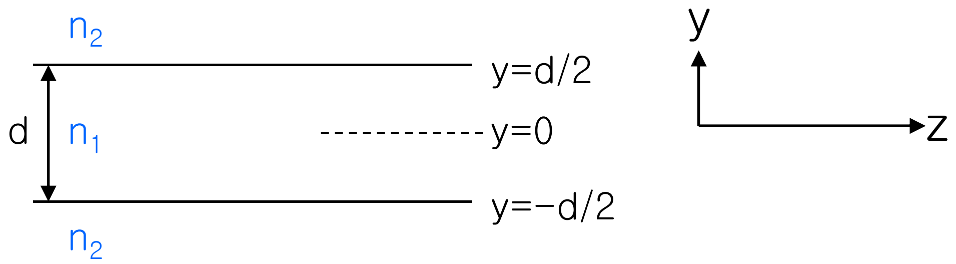
Waveguide Basics





Waveguide Basics

3-layer dielectric waveguide



Full analysis starting from wave equations.

$$\nabla^2 \overline{E}(y,z,t) = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}(y,z,t)$$

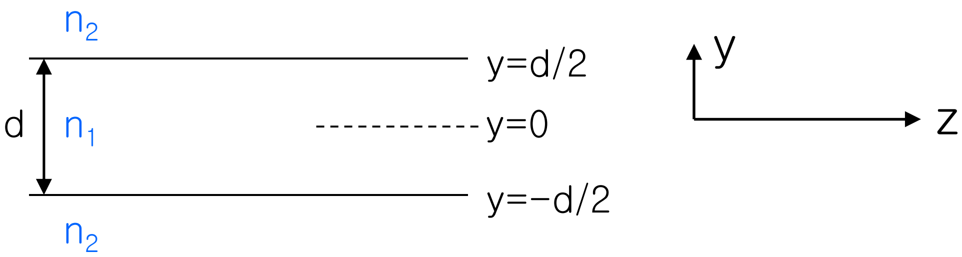
Assuming $\overline{E}(y,z,t) = \overline{E}(y,z) \cdot e^{j\omega t}$,

$$\nabla^2 \overline{E} + k^2(y)\overline{E} = 0, \text{ where } k^2(y) = \mu\varepsilon(y)\omega^2$$

$$k(y) = n_2 k_0 \text{ for } |y| > \frac{d}{2}; \text{ cladding}$$

$$k(y) = n_1 k_0 \text{ for } |y| < \frac{d}{2}; \text{ core}$$

Waveguide Basics



Consider TE Solution.

$$\overline{E}(y,z) = \overline{x}\ E(y)\ e^{-j\beta z}$$

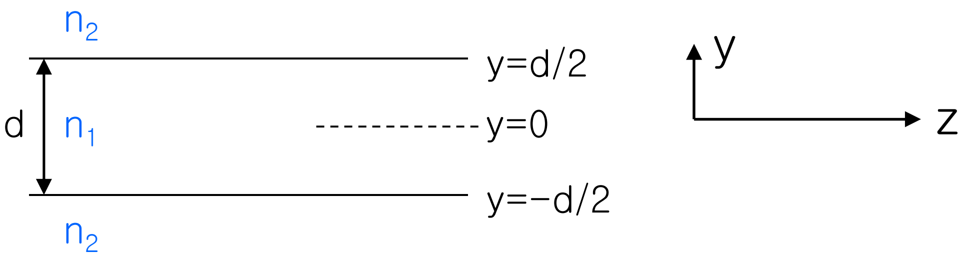
Then, $$\frac{d^2E(y)}{dy^2} + (k^2(y) - \beta^2)E(y) = 0$$

=> Eigen value equation. Solve for β and $E(y)$

$k^2(y) - \beta^2 > 0$ in core => $E(y) \sim \sin(k_y y)$ or $\cos(k_y y)$ with $k_y = \sqrt{(n_1 k_0)^2 - \beta^2}$

$k^2(y) - \beta^2 < 0$ in cladding => $E(y) \sim \exp(\alpha y)$ or $\exp(-\alpha y)$ with $\alpha = \sqrt{\beta^2 - (n_2 k_0)^2}$

Waveguide Basics



Solutions

$$y > \frac{d}{2} : E(y) = A\exp(\alpha y) + B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = C\sin(k_y y) + D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = E\exp(\alpha y) + F\exp(-\alpha y)$$

Here, A=0 and F=0.
For easy analysis, divide the solutions into even and odd solutions

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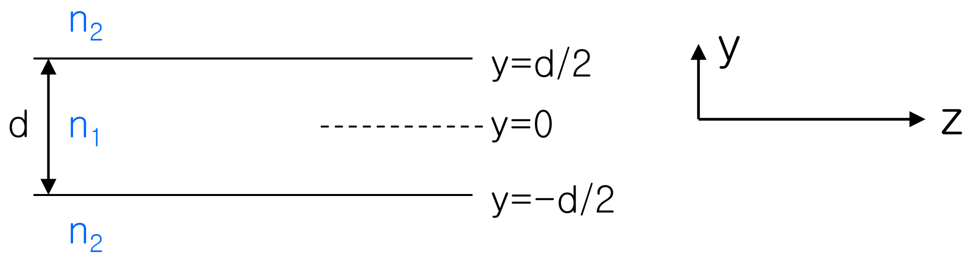
Waveguide Basics



Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$$(E = B)$$

Apply boundary conditions:

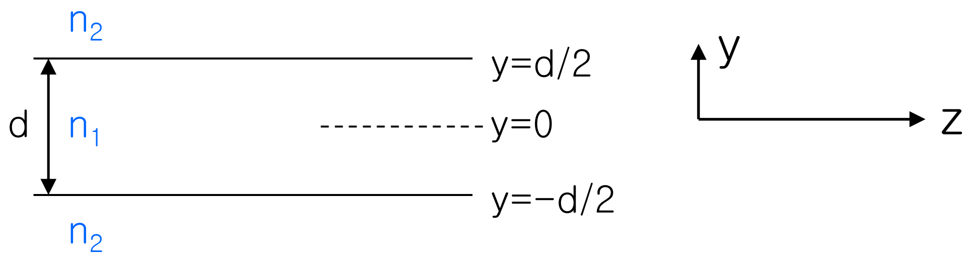
$E(y)$ and $\dfrac{dE(y)}{dy}$ are continuous at $y = \pm\dfrac{d}{2}$

$$B\exp(-\alpha\frac{d}{2}) = D\cos(k_y\frac{d}{2}) \quad \text{------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = -k_y D\sin(k_y\frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)} \Longrightarrow \alpha = k_y\tan(k_y\frac{d}{2})$$

Waveguide Basics



Odd Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$$

$$(E = -B)$$

Apply boundary conditions.

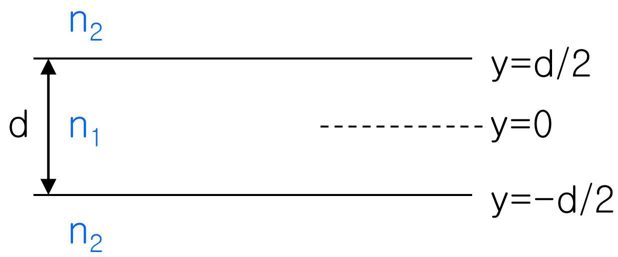
$E(y)$ and $\dfrac{dE(y)}{dy}$ are continuous at $y = \pm\dfrac{d}{2}$

$$B\exp(-\alpha\frac{d}{2}) = D\sin(k_y\frac{d}{2}) \quad \text{-------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = k_y D\cos(k_y\frac{d}{2}) \quad \text{------ (2)}$$

$$\frac{(2)}{(1)} \Longrightarrow \alpha = -k_y\cot(k_y\frac{d}{2}) = k_y\tan(k_y\frac{d}{2} - \frac{\pi}{2})$$

Waveguide Basics



Even: $\alpha = k_y \tan(k_y \frac{d}{2})$

What do these mean?

Odd: $\alpha = k_y \tan(k_y \frac{d}{2} - \frac{\pi}{2})$

Determine k_y and α that satisfy above conditions.

For graphical analysis, do following normalization.

Let $X = k_y \frac{d}{2}$, $Y = \alpha \frac{d}{2}$

Then, $Y = X \tan X$ for even

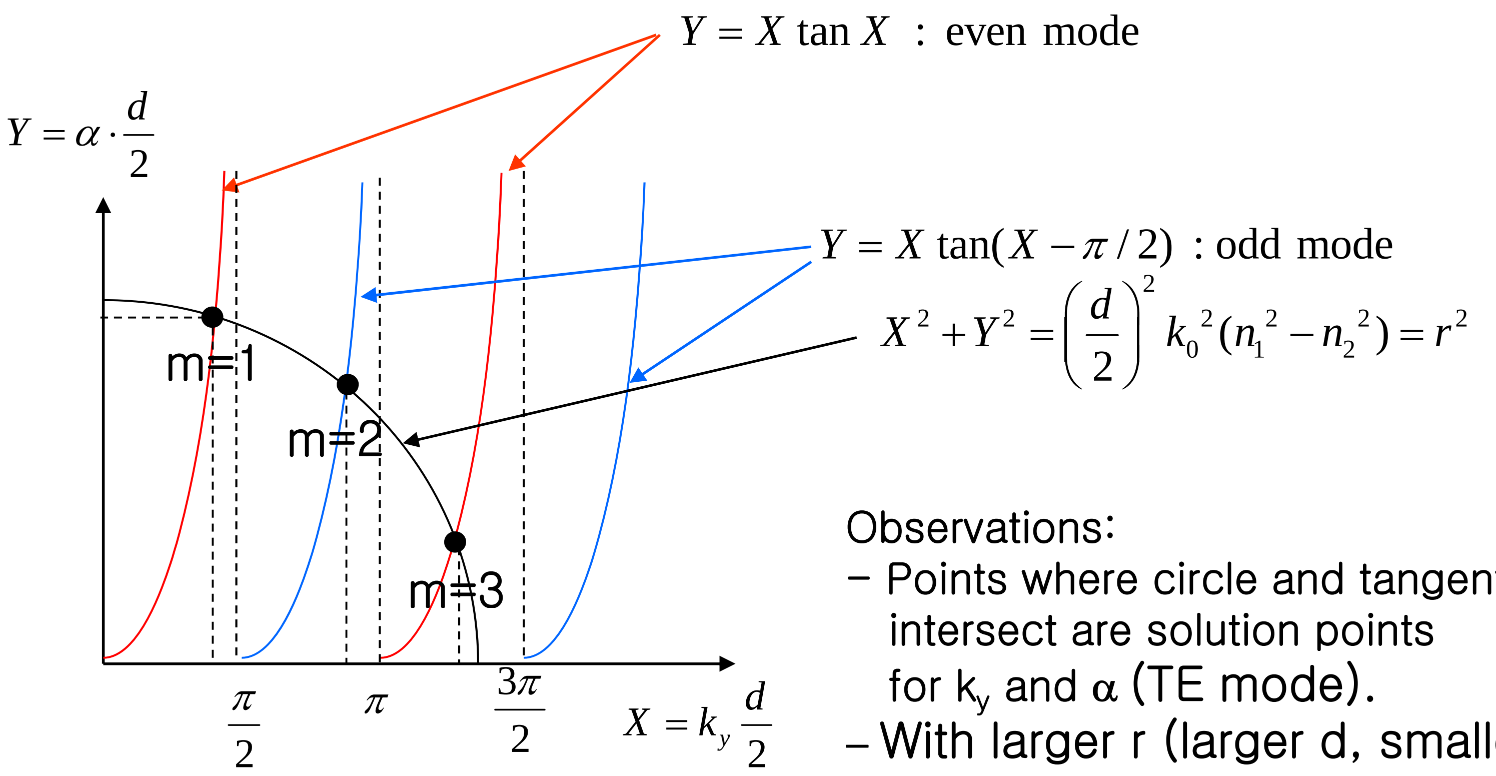
$Y = X \tan(X - \frac{\pi}{2})$ for odd

Plot these on X-Y plane.

But $X^2 + Y^2 = \left(\frac{d}{2}\right)^2 (k_y^2 + \alpha^2) = \left(\frac{d}{2}\right)^2 [(n_1 k_0)^2 - \beta^2 + \beta^2 - (n_2 k_0)^2]$

$= \left(\frac{d}{2}\right)^2 k_0^2 (n_1^2 - n_2^2) = r^2$

Waveguide Basics

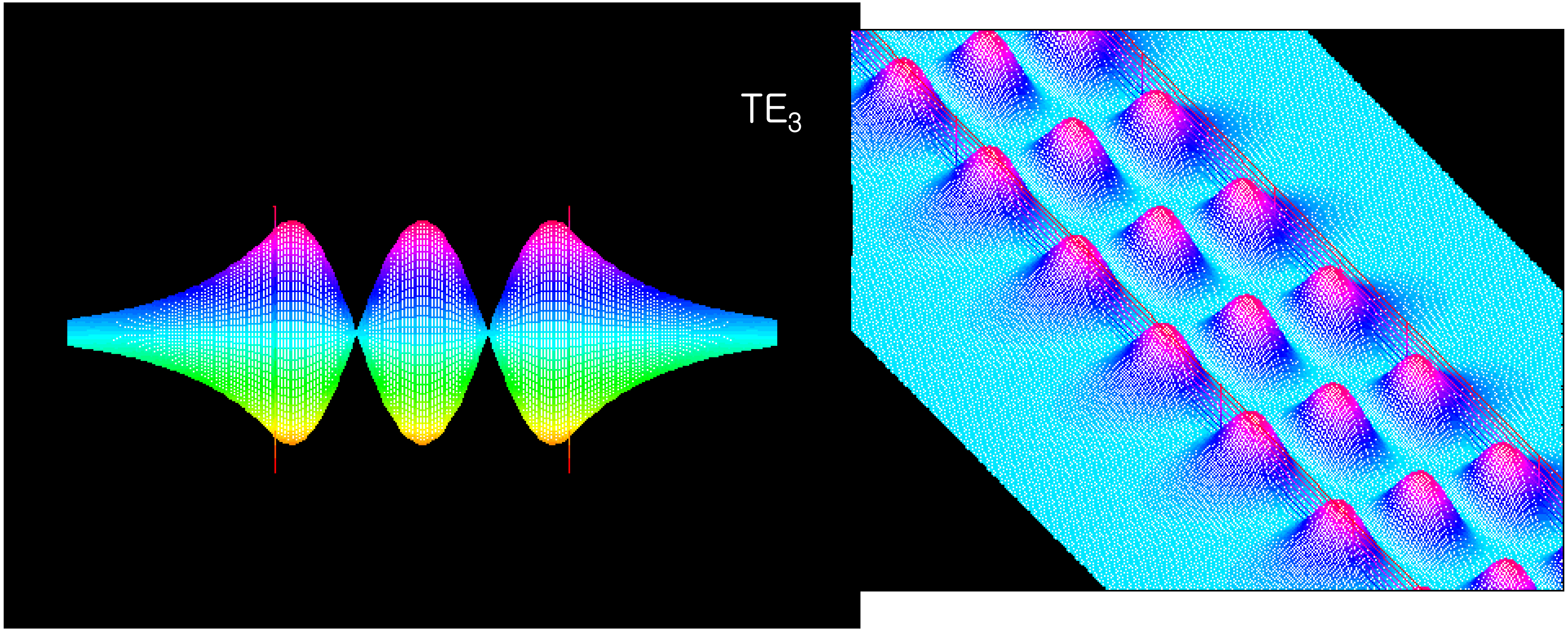


Observations:

- Points where circle and tangent curves intersect are solution points for k_y and α (TE mode).
- With larger r (larger d, smaller λ, larger $n_1^2 - n_2^2$), more modes exist.
- There is at least one even mode.
- Even, odd, even, odd ...

Waveguide Basics

E(y) profile: n_1=1.5, n_2=1.495, d=10μm, λ=1μm



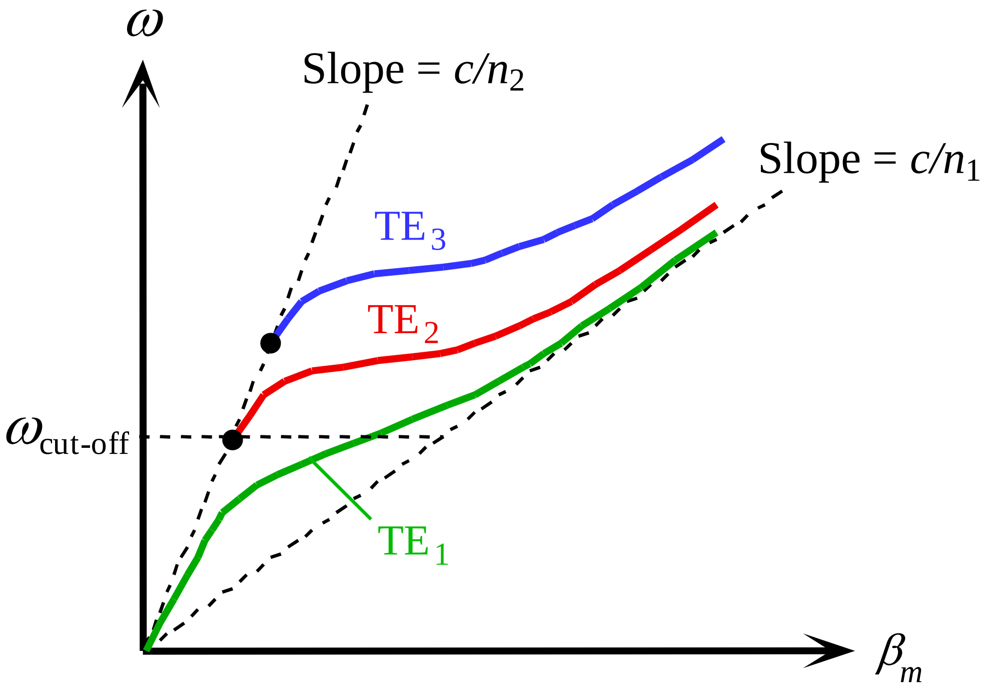
Waveguide Basics

Wave is not entirely confined within core: Confinement factor

$$\Gamma = \frac{\text{Power inside core}}{\text{Total Power}} = \frac{\int_{y=-\frac{d}{2}}^{y=\frac{d}{2}} |E(y)|^2 \, dy}{\int_{y=-\infty}^{y=\infty} |E(y)|^2 \, dy}$$

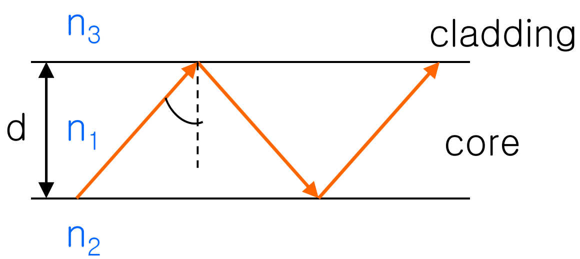
For higher modes, how does Γ change?

Waveguide Basics



Group velocities are different for different modes => modal dispersion
Need a single-mode waveguide in order to avoid signal distortion.
How do you design a single mode waveguide?

Waveguide Basics



$$V = k_0 d (n_1^2 - n_2^2)^{1/2}$$

(Normalized k)

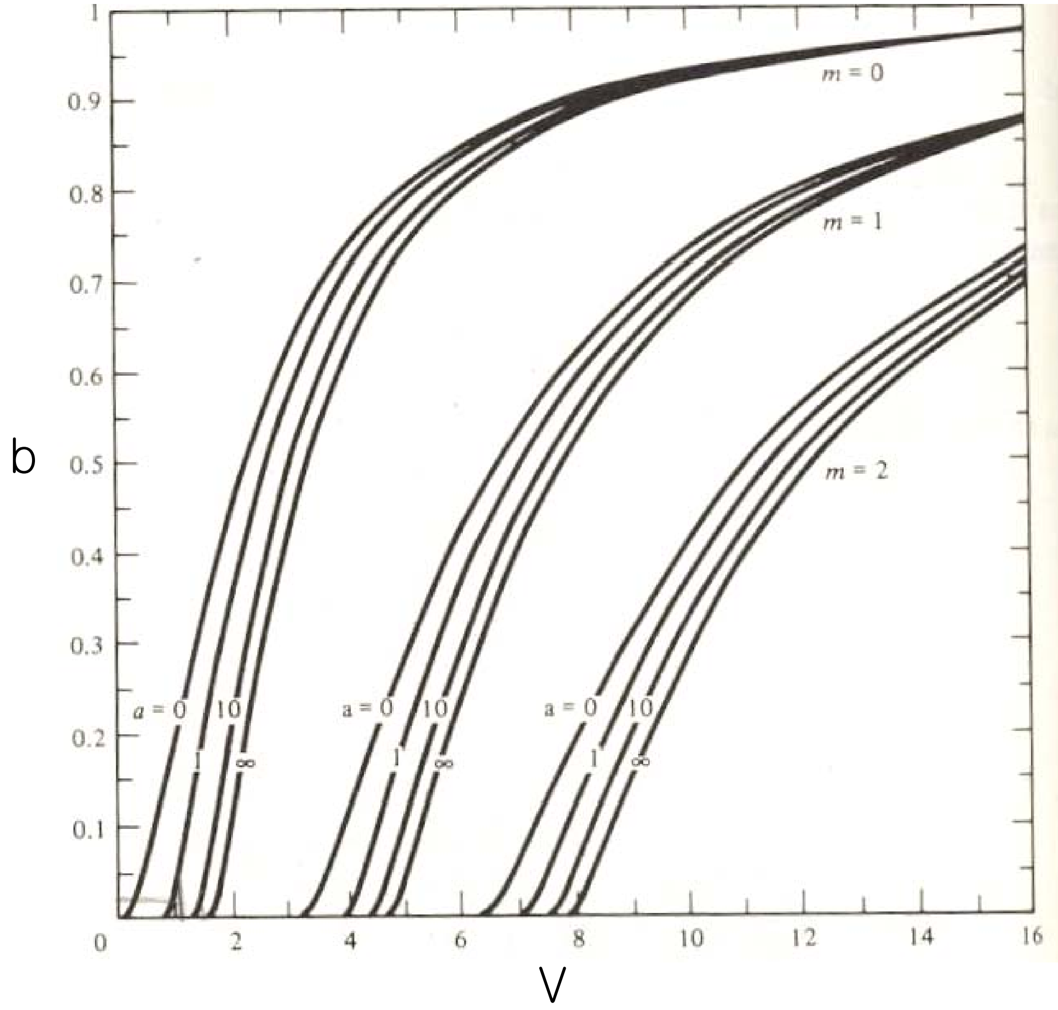
$$b = \frac{\left(\beta / k_0\right)^2 - n_2^2}{n_1^2 - n_2^2}$$

(Normalized β)

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}$$

(Asymmetry factor)

b-V diagram for TE mode



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Waveguide Basics

Issues for practical waveguides

- Precise control of dimension and refractive index
- Low loss at desired λ
- Mass production possible
- Integration desirable (Integrated Optics)
- Electrical control of refractive index (Electro-Optic effect)

Materials used for waveguides

- Silica ➔ Optical fiber
- Semiconductors: GaAlAs, InGaAsP, Si/SiO_2
- Dielectric materials: $LiNbO_3$ with Ti doping

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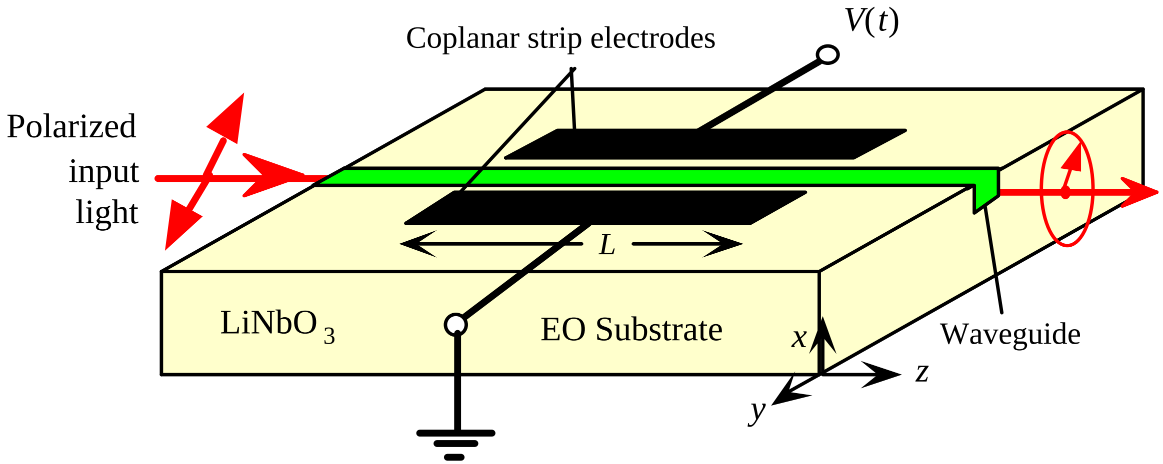
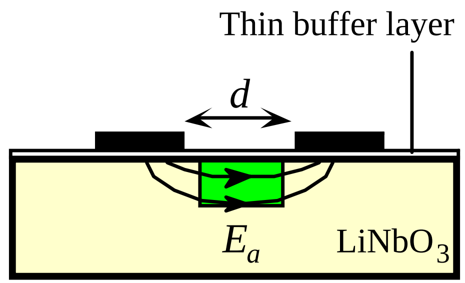
Waveguide Basics

LiNbO$_3$ waveguide

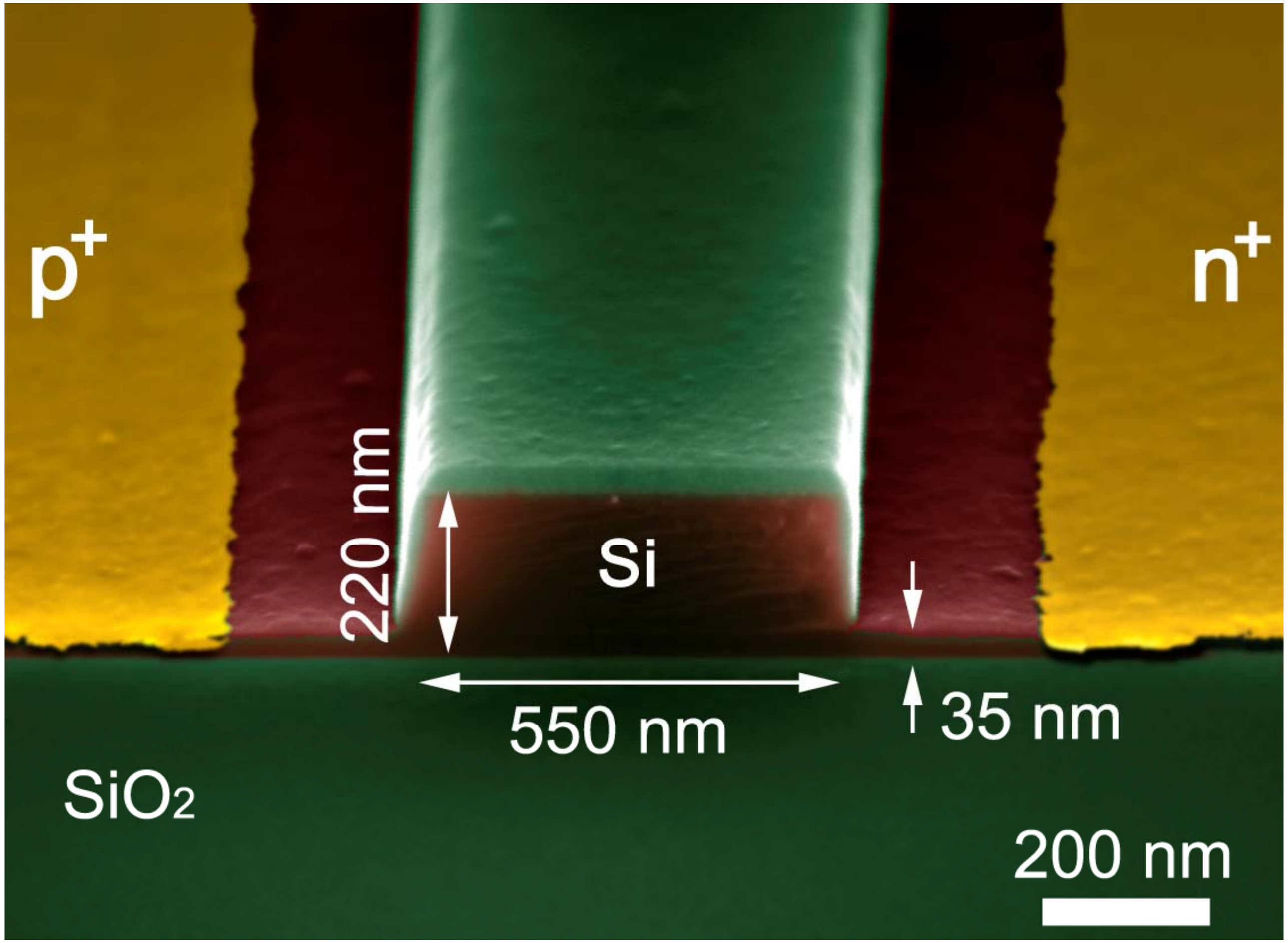




Cross-section

Waveguide Basics

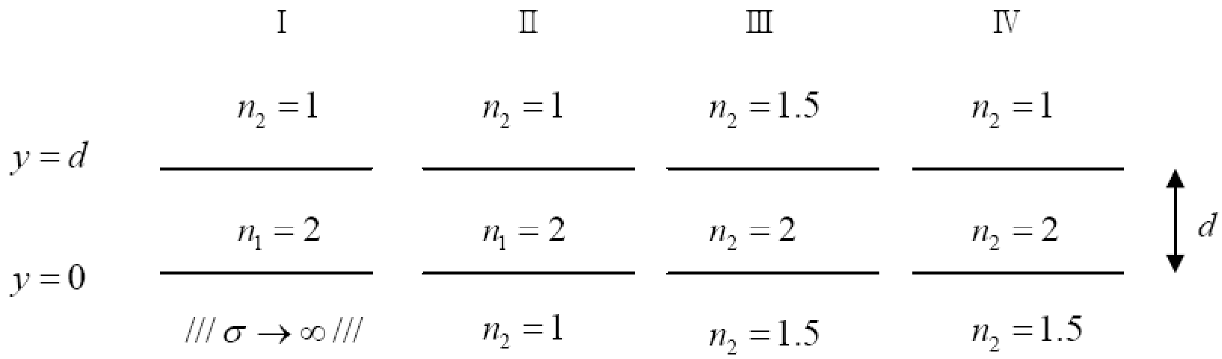
Si/SiO waveguide



Waveguide Basics

Problem 1

Several different types of waveguides having the same core material and thickness are shown below.



(a)(10) If we sketch the fundamental mode power distribution for each waveguide, which waveguide has the largest y value for the peak power position? Explain.

(b)(10) Among Type II, III, IV waveguides, which has the largest value for the fundamental mode effective index? Explain.

(c)(10) Among Type II, III, IV waveguides, which has the largest value for the fundamental mode confinement factor? Explain.

Waveguide Basics

Problem 2

(a)(10) From the b-V diagram provided separately, determine the wavelength range within which the fiber is a single-mode waveguide. Use n_1 (core reflective index)= 1.458, n_2 = 1.452, and a (core radius) = 3.5 μm.

(b)(10) In a three-layer symmetric dielectric waveguide with n_1 (core reflective index)= 1.458, n_2 = 1.452, and d (core thickness) = 7 μm, what is the wavelength range within which the waveguide has a single TE mode?

(c)(10) A singe mode fiber has loss of 0.2 dB/km at λ= 1.5 μm. If 1 mW of 1.5 μm light is introduced at the input, what is the output power at the end of 100km long fiber?

Waveguide Basics

Problem 3

A symmetric three-layer waveguide is shown below. Consider only TE mode for this problem.

(a) Determine how many modes the waveguide can support for λ=1.0μm.

(b) Sketch the electric field intensity in the waveguide for each mode.

(c) When λ is increased, the number of modes the waveguide can support may change. What is the largest wavelength for which the mode number remains the same as what was obtained in (a)?